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The two-dimensional site-diluted Ising model: a short-time-dynamics approach

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Abstract
The site-diluted Ising ferromagnet is investigated on a square lattice, within short-time-dynamics numerical simulations, for different site concentrations. The dynamical exponents θ and z are obtained and it is shown that these exponents do depend strongly on the disorder, exhibiting a clear breakdown of universality, characterized by relative variations of nearly 100% in the range of site concentrations investigated. In what concerns the static exponents β and ν, universality is preserved within the error bars.

1. Introduction

Although a large effort has been dedicated to the investigation of disordered magnetic systems, a lot of controversial aspects remain with no definitive answer. From the theoretical point of view, there exist only a few exact results, in such a way that approximation methods have to be employed. On the other hand, in the experimental context, most of the measurements undergo difficulties due to long relaxation times, which also appears to be a major obstacle in numerical simulations. Among these, disordered ferromagnets [1, 2] have faced the attention of many researchers, and a proper understanding of their behavior represents a great challenge in the physics of disordered systems. One is mostly concerned with the possible modifications, in the pure-system critical properties, due to the presence of randomness. According to the Harris criterion [3], the disorder will (will not) modify the critical behavior, i.e. the universality class, if in the corresponding pure system the specific-heat critical exponent α is positive (negative). Obviously, such a criterion is useless for the Ising ferromagnet in two dimensions, since $\alpha = 0$ for the pure case. As a consequence of this, disordered Ising ferromagnets in two dimensions have been the object of a lot of controversy, with different scenarios emerging. Several works support the *strong-universality* picture [2, 4–6], claiming that the presence of disorder affects the critical properties of the model only through logarithmic corrections to the pure-system behavior. It should be emphasized that such a picture is strictly valid only in the limit of weak disorder [4], since the disorder is treated as a perturbation. On the other hand, some numerical works [7–9] suggest thermodynamic quantities presenting the usual power-law behavior with the critical exponents varying continuously with the disorder, in such a way as to keep certain ratios (like γ/ν and β/ν) fixed at the pure-system values; this is the so-called *weak-universality* scenario [10]. One of the most investigated and controversial models is the two-dimensional site-diluted Ising ferromagnet; in this case, a systematic finite-size scaling analysis [11] claims that a clear discrimination between the strong- and weak-universality scenarios requires numerical simulations with much larger lattice sizes than the ones performed so far.

Numerical simulations in the short-time regime became an important tool in the investigation of critical phenomena [12]. The interesting point, in this so-called short-time-dynamics method, is that important scaling behavior seems to be already present in the early stages of the dynamical evolution of some statistical mechanics models at criticality [13]. One of the advantages of this procedure is that, in its early stage of evolution, the system presents small spatial and temporal correlation lengths. Therefore, apart from the smaller times that one is concerned with in these simulations, one gets

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a substantial reduction of finite-size effects, resulting in a considerable decrease of computational effort.

In the present work we investigate the ferromagnetic Ising model on a square lattice with dilution of sites (probability p for active sites and $(1-p)$ for diluted ones), in the short-time regime. Several values of site concentrations p are considered in the range $0.70 \leqslant p \leqslant 1.00$. The scaling of quantities, like the magnetization and some of its moments, as well as simple correlation functions, are considered within the short-time-dynamics procedure. For the dynamical critical exponents, a clear breakdown of universality is found, with exponents varying typically by a factor of two in the range of site concentrations considered. Due to the magnitude of variations in such exponents, as well as to the quality of the fits, it seems very unlikely that corrections to scaling may change this picture qualitatively. Contrary to that, the static critical exponents β and ν follow universality within the error bars. In the next section we define the numerical procedure and the model; in section 3 we present and discuss our results.

2. The numerical procedure and the model

Based on the simple scaling hypothesis for the moments of the magnetization, several interesting power-law behaviors have been predicted, and verified numerically, in a wide variety of pure magnetic systems, at criticality, for a short time t greater than a certain microscopic timescale $t_{\rm mic}$ [12]. In the case of Ising variables, $S_i(t) = \pm 1$, one may represent the kth moment of the magnetization at time t as

$$M^{(k)}(t) = \frac{1}{N^k}\left\langle \left(\sum_{i=1}^{N} S_i(t) \right)^k \right\rangle, \tag{1}$$

where N stands for the total number of spins of the lattice and $\langle \cdots \rangle$ represents an average over different samples at time t. For pure systems, this corresponds to an average over distinct sequences of random numbers up to time t, whereas in the case of disordered systems it includes, as well, an additional average over different disorder realizations. Since we are dealing with an out-of-equilibrium regime, the scaling behaviors should depend on the initial conditions, which, for ferromagnetic systems, are essentially taken into account through the definition of the initial magnetization of the system. If the system is quenched from an initial state with a small magnetization, $m_0 \ll 1$, one gets for the magnetization at time t (herein we shall use the standard notation, $M^{(1)}(t) \equiv M(t)$), its second moment, and the two-time autocorrelation function, respectively:

$$M(t) \sim m_0 t^{\theta}, \tag{2}$$

$$M^{(2)}(t) \sim t^{(d-2\beta/\nu)/z}, \tag{3}$$

$$C(t) = \frac{1}{N}\left\langle \sum_{i=1}^{N} S_i(t) S_i(0) \right\rangle \sim t^{-(d/z)+\theta}, \tag{4}$$

where d corresponds to the lattice dimension. In the scaling laws above, the standard equilibrium exponents β and ν, as well as the dynamic exponent z, appear, leading to the possibility of evaluating them already for very short times of the evolution of the system. However, a new and independent dynamical exponent appears, θ, which is related to the increase of the magnetization at the critical temperature, when the system is quenched from a high-temperature state. In order to compute θ from equation (2) one should consider distinct, small (although finite) values of m_0, and then take an extrapolation to $m_0 \to 0$. An equivalent way to carry on such a procedure consists in starting with a random initial configuration (i.e. magnetization and correlation length both equal to zero) and computing the correlation function [14]:

$$Q(t) = \frac{1}{N}\left\langle \sum_{i,j=1}^{N} S_i(t) S_j(0) \right\rangle \sim t^{\theta}. \tag{5}$$

Now, if one starts the system with a completely ordered configuration ($m_0 = 1$), the magnetization should follow the simple power-law behavior:

$$M(t) \sim t^{-\beta/(\nu z)}. \tag{6}$$

Within this initial condition one may compute also the absolute value of the derivative with respect to the temperature, at the critical temperature $T_{\rm c}$ [15, 16]:

$$D(t) = \left| \frac{\partial \ln M(t,\tau)}{\partial \tau} \right|_{\tau=0}, \tag{7}$$

where $\tau = (T - T_{\rm c})/T_{\rm c}$. This derivative may be computed numerically through

$$D(t) = \lim_{\delta \to 0} \frac{1}{\delta} |\ln M(t,0) - \ln M(t,-\delta)|, \tag{8}$$

and is expected to present the following behavior:

$$D(t) \sim t^{1/(\nu z)}. \tag{9}$$

In addition to that, the ratio of moments:

$$U_2(t,L) = \frac{M^{(2)}(t,L)}{[M(t,L)]^2} - 1, \tag{10}$$

should behave like $U_2(t, L \to \infty) \sim t^{d/z}$, from which one could compute the exponent z independently. However, the quantity $U_2(t,L)$ has not been very successful when applied to simple systems, like, for example, the two-dimensional $q = 3$ Potts model [12]. The reason for such a failure is probably related to the scaling behavior of the second moment $M^{(2)}$ with the initial condition $m_0 = 1$. Therefore, a new quantity has been proposed, which seems to work better than the one defined in equation (10); now, one considers mixed initial conditions to get [17, 18]

$$F_2(t) = \frac{M^{(2)}(t,L)|_{m_0=0}}{[M(t,L)]^2|_{m_0=1}} \sim t^{d/z}. \tag{11}$$

In principle, one can make use of these scaling laws in order to find all the exponents defined above; notice that there are more scaling laws than exponents, in such a way that computing a given exponent through different sets of scaling forms should

lead to the same results, within the error bars associated with the numerical procedure. This framework has worked fairly well in the study of pure systems, with a few exceptions [12].

The validity of the scaling hypothesis for disordered systems is very controversial, even for equilibrium properties, leading to well-known proposals of corrections to scaling [2, 4–6]. The introduction of disorder tends to smooth out the usual sharp scaling behavior in the thermodynamic quantities near criticality, in such a way that the proposal of scaling laws becomes troublesome. On the other hand, the numerical results from computer simulations are characterized by an increase in the fluctuations (i.e. larger error bars) in the quantities considered, reducing the reliability in the corresponding critical-exponent estimates. Similar problems are expected to appear within the short-time-dynamics framework, as verified in a few applications [19–21]; some examples of such difficulties are listed below.

(i) Severe (in fact, power-law) corrections to scaling were proposed for the second moment of the magnetization and two-spin autocorrelation function (cf equations (3) and (4)) in the analysis of the two-dimensional random bond Ising model [20].

(ii) Different estimates for the same exponents, through the analysis of the scaling properties of distinct quantities, were found in the study of the two-dimensional bond-diluted Ising model [21]. In this case, the authors made use of the scaling relations in equations (2)–(4), (6) and (10), as well as the moments ratio, $U_4(t, L) = 1 - [M^{(4)}(t, L)/(3M^2(t, L))]$, this later quantity being evaluated with $m_0 = 0$. This suggests that some of the scaling laws employed in [21] may not be appropriate for the investigation of this problem. It is possible that the same troublesome scaling laws of case (i), i.e. equations (3) and (4), as well as the moments ratio of equation (10) (which has already failed for simpler problems [12]) may be the cause of such conflicting results. Apart from this, an accurate estimate of the exponent z through the analysis of the moments' ratio $U_4(t, L)$ seems to be hard for disordered systems, since one should consider a scaling collapse from two-time series, which usually present more fluctuations than those of a pure system.

Due to the difficulties mentioned above, we shall consider only a restricted set of quantities, from those defined above, in order to evaluate critical exponents for a similar disordered magnetic system, as described below.

Herein, we shall investigate the nearest-neighbor interaction site-diluted ferromagnetic Ising model, defined through the Hamiltonian

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \varepsilon_i \varepsilon_j S_i S_j, \tag{12}$$

with $J > 0$, and $S_i = \pm 1$. The model is considered on a square lattice of linear size L, with a quenched site disorder following the probability distribution

$$P(\varepsilon_i) = p\delta(\varepsilon_i - 1) + (1 - p)\delta(\varepsilon_i). \tag{13}$$

In the limit $L \to \infty$, it exhibits a well-known paramagnetic–ferromagnetic phase transition at a critical temperature $T_c(p)$, which decreases for decreasing values of p in the range $p_c \leqslant p \leqslant 1$ ($p_c \approx 0.59$).

It is well known that, in the thermodynamic limit, there is a single percolating cluster composed of an infinite number of active sites for $p > p_c$; all remaining clusters are finite and do not contribute to the averages. For $p < p_c$, there is no infinite cluster, in such a way that there is no finite contribution for thermodynamic quantities, like, for example, the magnetization. However, for finite systems, large fluctuations may appear due to the fact that one may find samples with no percolating cluster for $p > p_c$, as well as samples characterized by a cluster that spans through the whole lattice, for $p < p_c$. Therefore, it is important to identify, in each sample, the presence of a percolating cluster [22] in order to perform the corresponding spin updating procedure.

The spins on each active site i of the percolating cluster were updated according to a Glauber dynamics, in which one associates a probability $p_i(t)$, at time t:

$$p_i(t) = \frac{1}{1 + \exp[-2\beta J \sum_j S_j(t)]}, \tag{14}$$

where the summation $\sum_j S_j(t)$ applies to nearest-neighbor spins on active sites. As usual, a uniform random number, $0 \leqslant z_i(t) \leqslant 1$, is generated in order to be compared with the corresponding flipping probability $p_i(t)$. In the Glauber dynamics, the value of the new variable $S_i(t + 1)$ depends also on $S_i(t)$ and is defined as

$$S_i(t+1) = \begin{cases} 1 & \text{if } z_i(t) \leqslant p_i(t) \\ -1 & \text{if } z_i(t) > p_i(t) \end{cases} \quad \text{when } S_i(t) = -1, \tag{15a}$$

$$S_i(t+1) = \begin{cases} -1 & \text{if } z_i(t) \leqslant 1 - p_i(t) \\ 1 & \text{if } z_i(t) > 1 - p_i(t) \end{cases} \quad \text{when } S_i(t) = 1. \tag{15b}$$

In section 3 we present and discuss our results.

3. Results and discussion

The model was studied within the short-time-dynamics procedure, as described above, with seven different site concentrations, $p = 0.70, 0.75, 0.80, \ldots, 1.00$, for which the critical temperatures $T_c(p)$ were already computed, to a high degree of accuracy, in the literature (see, e.g., [23–27]). In the analysis that follows we have used the recent critical-temperature estimates of [27], which were obtained through an extrapolation to the thermodynamic limit taking into account a logarithmic correction on the finite-size dependent pseudo-critical temperature.

We have applied the algorithm of Hoshen–Kopelman [22, 28] for an appropriate identification of the percolating cluster in each sample. In our computations we have considered N as the number of sites of the percolating cluster, which varied from sample to sample. In addition to that, averages were carried only over samples presenting a percolating cluster. Our unit of time in the present problem

Table 1. Estimates of the critical exponents θ, z, β, and ν, for the two-dimensional site-diluted ferromagnetic Ising model, from short-time-dynamics numerical simulations, at criticality. The critical-temperature estimates, $T_c(p)/T_c(1)$, shown above, were taken from [27], where $T_c(1)$ refers to the well-known exact critical temperature of the two-dimensional Ising model on a square lattice, $k_B T_c(1)/J = 2.269\,185\ldots$. The exponents θ were calculated by using both equations (2) and (5) and were found to agree, within the error bars; the results above correspond to an average value of these two estimates, taking into account their respective error bars. The combination of exponents $\beta/(\nu z)$ and the dynamical exponent z were computed from equations (6) and (11), respectively, which lead to the estimates for the ratios β/ν.

p	$T_c(p)/T_c(1)$	θ	$\beta/(\nu z)$	z	β/ν	β	ν
0.70	0.4728	0.103(6)	0.0329(4)	3.78(6)	0.124(2)	0.129(7)	1.04(4)
0.75	0.5724	0.127(5)	0.0365(4)	3.37(5)	0.123(3)	0.129(9)	1.05(5)
0.80	0.6655	0.137(5)	0.0413(3)	2.95(4)	0.122(3)	0.123(8)	1.01(4)
0.85	0.7538	0.158(3)	0.0456(3)	2.72(4)	0.124(3)	0.126(7)	1.02(3)
0.90	0.8392	0.167(4)	0.0486(3)	2.51(4)	0.122(3)	0.122(5)	1.00(2)
0.95	0.9210	0.180(3)	0.0526(2)	2.33(3)	0.123(2)	0.122(4)	0.99(2)
1.00	1.0000	0.194(2)	0.0590(2)	2.14(3)	0.126(2)	0.123(4)	0.98(2)

corresponds to a complete sweep of the percolating cluster, denoted from now on by 1 Monte Carlo (MC) step.

Due to the difficulties found in previous studies of disordered magnetic systems [20, 21] (see items (i) and (ii) discussed in section 2), we have restricted our analysis only to the set of scaling laws in equations (2), (5), (6), (9) and (11), which, in our view, correspond to the most reliable ones for the study of the site-diluted Ising model. We have found good linear fits from these equations, when considered in the critical temperatures $T_c(p)$ of [27], although small variations in such critical-temperature estimates (typically, in the fourth decimal digits) yielded equally good fits. Our critical-exponent estimates were obtained by choosing linear fits over time intervals leading to the highest fit quality. Such time intervals depended on the quantity to be analyzed, and they were adjusted in each case such as to yield linear-fit qualities with a correlation parameter Q not less than 0.98, i.e. $0.98 \leqslant Q \leqslant 1.00$. Our simulations were carried for lattice sizes $L = 128$ and 256, with averages over the fraction of samples presenting a percolating cluster being performed among a number of different samples (disorder configurations) of the order of 10^5. Although our linear fits were considered over time intervals leading to the highest fit qualities, the error bars were computed in such a way as to cover estimates from different time ranges; for the dynamic critical exponents, these error bars also cover the results for the two lattice sizes considered. Therefore, in what concerns the dynamic critical exponents θ and z, our estimates for the two lattice sizes coincided, within the error bars, whereas for the static exponents β and ν the results for the larger size appeared to be more reliable.

In figure 1 we exhibit the short-time magnetization growth, for an initial magnetization $m_0 = 0.01$, obtained from simulations on a square lattice of linear size $L = 128$ with different site concentrations p. In each case, the system is quenched from a high-temperature state, at its corresponding critical temperature $T_c(p)$ (according to the values estimated in [27], as shown in table 1). The behavior presented suggests the validity of the scaling law in equation (2), with positive values of θ, for typical time intervals $10 \leqslant t \leqslant 100$ (in MC steps). However, one observes that $M(t)$ grows faster for higher values of p. Moreover, the data for $p = 0.90 \rightarrow 1.00$ (weaker disorder) are almost superposed, whereas for $p = 0.70 \rightarrow 0.85$ (stronger disorder), the data are clearly distinguishable. A slower growth in the magnetization, for increasing disorder, reflects the presence of clusters of diluted sites, which contribute to decrease the connectivity of the system.

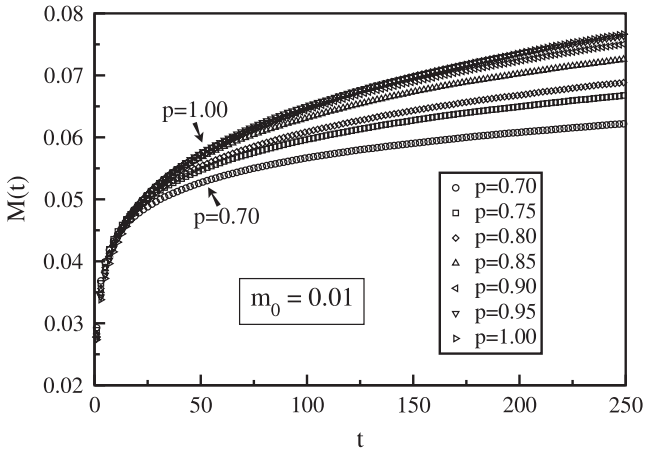

Figure 1. The magnetization $M(t)$ versus time t (in MC steps), for different site concentrations p at criticality, with an initial magnetization $m_0 = 0.01$.

The plot of $M(t)$ versus time, in logarithmic scale, is exhibited in figure 2 for the case $p = 0.70$ at criticality, with four different small values of the initial magnetization per site m_0. One observes a good agreement with the scaling law of equation (2), with precise straight lines in the time interval from $t = 10 \rightarrow t = 150$ MC steps. For each choice of m_0, the corresponding values of θ were estimated, by adjusting, with small variations in this time window, the time interval leading to the best linear fits; we have found a slow increase in the values of θ for decreasing values of m_0, as specified in figure 2. A linear extrapolation of these values leads, in the limit $m_0 \rightarrow 0$, to the estimate $\theta = 0.106(3)$. An equivalent estimate for the exponent θ may be carried through the correlation function of equation (5) by starting the system at criticality with an initial random spin configuration. In this case, one gets $\theta = 0.100(3)$, which agrees within the error bars with the estimate of the extrapolation $m_0 \rightarrow 0$. Therefore, the result in table 1 corresponds to an average between these two estimates, including their respective error bars. Similar

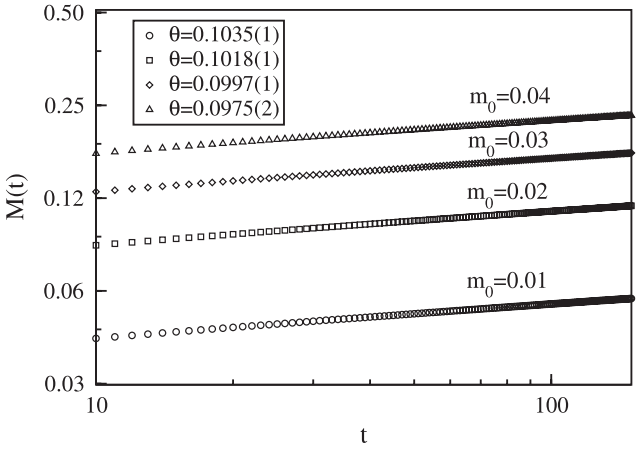

Figure 2. Logarithmic-scale (base 10) plots of the magnetization versus time t (in MC steps), in the case $p = 0.70$ at criticality, for different small values of m_0, according to equation (2).

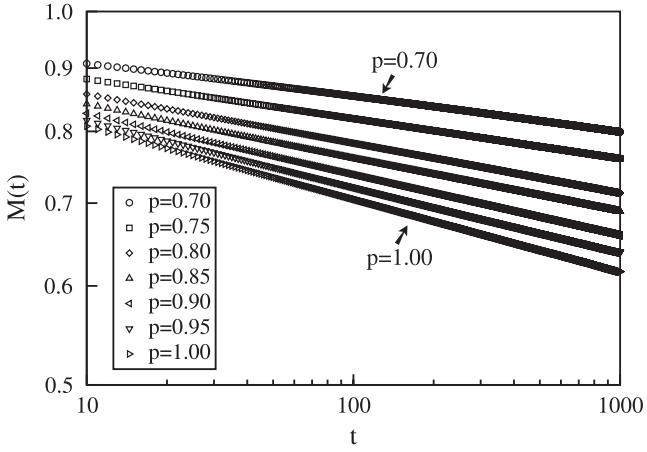

Figure 3. Logarithmic-scale (base 10) plots of the magnetization versus time t (in MC steps), for different site concentrations at criticality, from a completely ordered initial spin configuration, $m_0 = 1$ (see equation (6)).

procedures have been carried for the other values of p shown in table 1, and in all cases considered we have found an agreement, within the error bars, between the estimates of these two methods. We have observed a breakdown of universality in the results of the exponent θ, for varying values of p, in such a way that for $p = 0.70$ one obtains an estimate which is nearly half of the value for $p = 1.00$; it should be emphasized that our estimate for the exponent θ at $p = 1.00$ [$\theta = 0.194(2)$] is in agreement with well-known pure-system estimates, i.e. $\theta = 0.191(3)$ [12, 29]. Due to the magnitude of this breakdown of the universality effect, as well as to the quality of the fits, taking into consideration the good agreement between the two independent methods employed herein, it is very unlikely that corrections to scaling may change this picture qualitatively.

In figure 3 we exhibit the magnetization decay with time, in logarithmic scale, for several site concentrations at their corresponding critical temperatures (see table 1), produced by a quenching from a low-temperature configuration ($m_0 = 1$) with simulations on a square lattice of linear size $L = 256$. Once again, the quality of the straight-line fits show the validity of the scaling law of equation (6), for the present problem, in the time interval from $t = 10 \rightarrow t = 1000$ MC steps. The estimates of $\beta/(\nu z)$ were obtained from the slopes of the best linear fits, considering small variations in this time window. The results, presented in table 1, show a clear breakdown of universality in the quantity $\beta/(\nu z)$, which almost doubles its value as one goes from $p = 0.70$ up to $p = 1.00$. An important question concerns which exponents, within the combination $\beta/(\nu z)$, are responsible for this breakdown of universality.

The increase of the quantity $D(t)$ with time (cf equations (7)–(9)) is exhibited in logarithmic scale in figure 4, for simulations on a square lattice of linear size $L = 256$ with the site concentrations investigated. The results presented in figure 4 were computed from equation (8), at the critical temperatures presented in table 1, for each value of p. Although the corresponding derivatives may depend on the particular choice for the parameter δ, we have verified that our results did not change significantly (within the error bars) for $\delta < 0.001$; the results of figure 4 correspond to $\delta = 0.001$. The slopes of the straight lines yield the combination of exponents, $1/(\nu z)$, which varies typically in the range from 0.25 (for $p = 0.70$) up to 0.48 (for $p = 1.0$). Hence, the combination of exponents $1/(\nu z)$ is characterized also by a significant breakdown of universality.

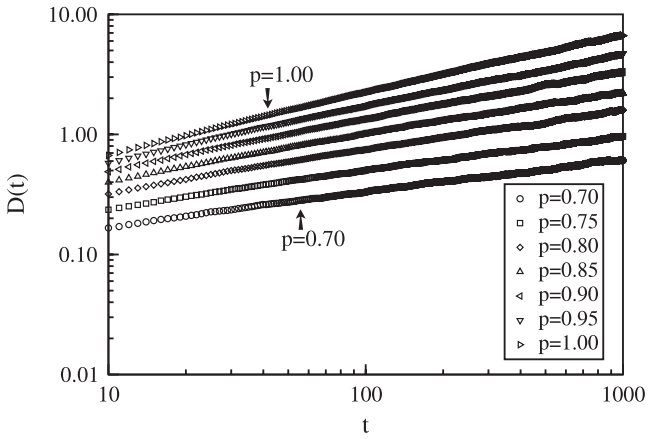

Figure 4. Logarithmic-scale (base 10) plots of the quantity $D(t)$ (cf equation (7)) versus time t (in MC steps) for different site concentrations at criticality. The slope of each straight line yields the exponent $1/(\nu z)$.

The ratio of moments of the magnetization, using mixed initial conditions, as defined in equation (11), is exhibited in figure 5 for different site concentrations at criticality (lattice size $L = 256$). The validity of the scaling law of equation (11) is verified through the good quality of the straight-line fits shown in the time interval from $t = 10 \rightarrow t = 1000$ MC steps. The slopes of the straight lines, which were computed, in each case, by considering small variations in this time window, correspond to the exponent $d/z \equiv 2/z$. From these plots one may estimate the dynamical critical exponent z, independently. The lines of figure 5 yield a clear breakdown of universality in the exponent z, which varies from its well-known value $z = 2.14(3)$, for $p = 1$ (close to the precise estimate $z = 2.1665 \pm 0.0012$ of [30], considering the error bars) up

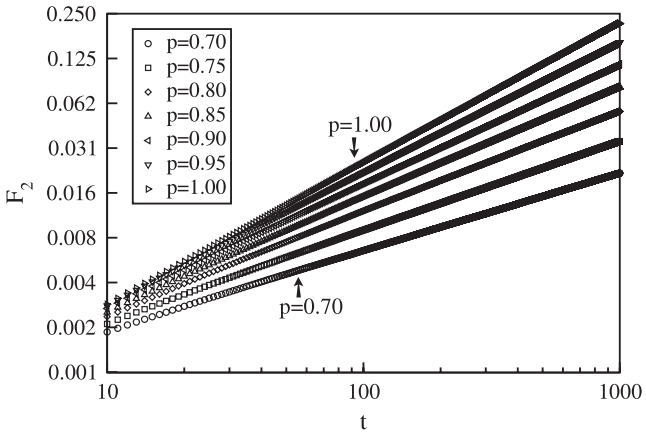

Figure 5. Logarithmic-scale (base 10) plots of the ratio of magnetization moments (see equation (11)) versus time t (in MC steps) for different site concentrations at criticality, using mixed initial conditions for the spins. The slope of each straight line yields the exponent $d/z \equiv 2/z$.

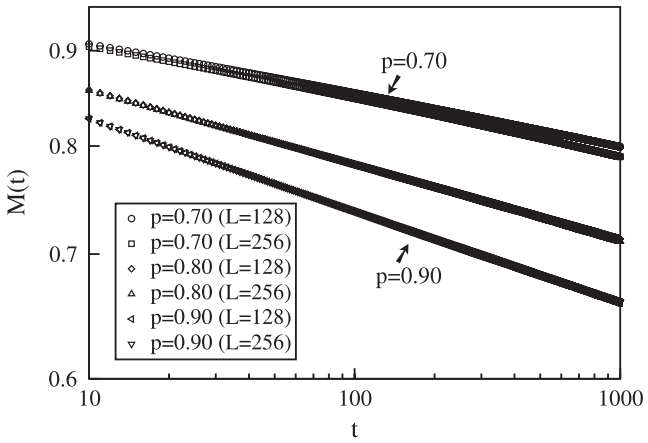

Figure 6. Logarithmic-scale (base 10) plots of the magnetization versus time t (in MC steps), for typical site concentrations at criticality, from a completely ordered initial spin configuration, $m_0 = 1$ (see equation (6)), are compared for the two lattice sizes studied, namely $L = 128$ and 256. It should be noticed that the data for $p = 0.80$ and 0.90 appear superposed.

to $z = 3.78(6)$, in the case $p = 0.70$, as shown in the fifth column of table 1. The exponent z, related to the divergence of the correlation time near criticality, increases for increasing site dilutions, as expected, reflecting an enhancement in the critical slowing down for smaller values of p.

Now, considering the estimates of the exponent z, one may use the results from the linear fits of figures 3 and 4 in order to get the static exponents β and ν. The resulting static exponents presented larger fluctuations for the lattice size $L = 128$, suggesting in some cases a slight breakdown of universality, which is attributed to finite-size effects. In fact, by increasing the lattice size ($L = 256$), universality is obtained within the error bars. These results, which are in agreement with those of recent Monte Carlo simulations [27], favor the strong-universality picture for static critical exponents of disordered magnetic systems, according to which the leading contributions for these exponents are universal and possible universality-breakdown contributions come only as logarithmic corrections.

In figures 6 and 7 we compare the results of simulations for the two lattice sizes studied herein, i.e. $L = 128$ and 256. It should be mentioned that the finite-size effects are very important in the present model, being responsible for the controversies concerning its static exponents and that these effects become more pronounced for higher site dilutions. In figure 6 we exhibit the magnetization decay, according to equation (6); one observes that for larger site concentrations ($p = 0.90$ and 0.80), the finite-size effects are negligible and the two lattice sizes considered lead essentially to the same results, considering the error bars. However, finite-size effects appear in figure 6 for the concentration $p = 0.70$, where one gets a small discrepancy between the results of the two lattice sizes. In fact, the results for $p = 0.70$ in the smaller lattice present a slight breakdown of universality in the ratio β/ν, which disappears for the larger lattice size. Comparisons of the results for the ratio of moments of equation (11), for the two lattice sizes, are shown in figure 7(a) ($p = 0.70$) and figure 7(b) ($p = 0.90$). One sees that the amplitude coefficient of $F_2(t)$ depends on the lattice size L; however, in what concerns its exponent, the finite-size effects are not significant, showing that the lattice size $L = 128$ is sufficient for the estimate of the dynamical exponent z. The plots shown in figures 6 and 7 suggest that the finite-size effects become more important for smaller site concentrations and for those quantities depending on static critical exponents.

The present results, associated with a breakdown of universality in the dynamic critical exponents, corroborate those found previously on a similar disordered system, namely the two-dimensional random bond Ising model [20]. In [20], the authors studied a disordered Ising ferromagnet defined in terms of a Hamiltonian analogous to the one of equation (12) by replacing $J\varepsilon_i\varepsilon_j \rightarrow J_{ij}$, where the couplings $\{J_{ij}\}$ were taken as J or rJ ($0 \leqslant r \leqslant 1$) randomly with probability 1/2. In what concerns universality, the site dilution, characterized by the probability p (analyzed herein) and the bond disorder, represented by the ratio of weak to strong bond intensity (considered in [20]), should lead to equivalent effects, at least qualitatively.

To conclude, we have studied the site-diluted ferromagnetic Ising model on a square lattice within short-time-dynamics simulations. Different values of site concentrations p were considered in the range $0.70 \leqslant p \leqslant 1.00$. Through an analysis of the scaling with time of the magnetization and some of its moments, as well as of a two-time correlation function, dynamical exponents and the static exponents β and ν were obtained, at criticality. An evident breakdown of universality was found in the dynamical critical exponents, which exhibited typical variations by a factor of two, in the range of site concentrations considered. However, in what concerns static critical exponents, our estimates preserve universality; this result is in agreement with previous standard Monte Carlo analysis [27], favoring the strong-universality picture for the static critical exponents of disordered magnetic systems, which claims that the presence of disorder affects

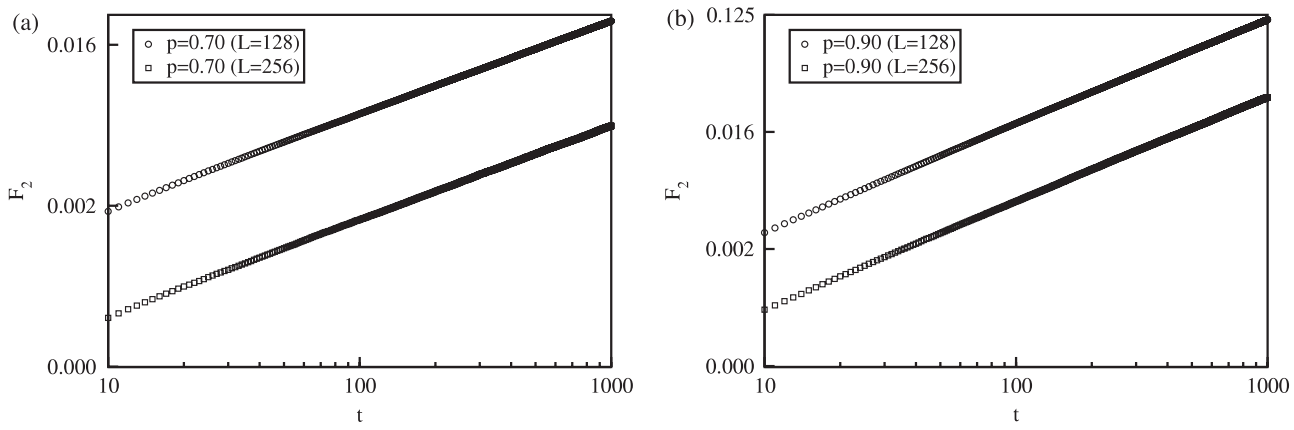

Figure 7. Logarithmic-scale (base 10) plots of the ratio of magnetization moments (see equation (11)) versus time t (in MC steps), for two typical site concentrations, $p = 0.70$ (a) and $p = 0.90$ (b), are compared for the two lattice sizes studied, namely $L = 128$ and 256.

the critical properties of the model only through logarithmic corrections to the pure-system behavior.

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